

## Quantum-fluctuation-induced spatial stochastic resonance at zero temperature

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We consider a model in which the quantum fluctuation can be controlled and show that the system responds to a spatially periodic external field at zero temperature. This signifies the occurrence of spatial stochastic resonance where the fluctuations are purely quantum in nature. Various features of the phenomenon are discussed.

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The phenomenon of stochastic resonance (SR) is precisely the enhancement of response to an external field with the help of noise. SR is manifested in bistable systems, where the fluctuations can drive the system from one energy minimum to the other. As a function of the noise, the response typically shows a maximum value at resonance. The basic features required to observe stochastic resonance are simple: an energy barrier, a weak external periodic signal, and a noise source. SR, which is observed in many natural phenomena, has therefore important applications in a variety of research areas, including nonlinear optics, solid state devices, and even neurophysiology [1].

Although most of the studies consider classical systems, stochastic resonance in quantum systems have attracted a lot of attention recently [1–4]. In the quantum systems, the quantum mechanical tunneling provides an additional channel to overcome a potential barrier. At exactly zero temperature, some numerical results for the tunneling phenomena are available for the periodically driven quantum double well system [2]. The noise is purely quantum in nature at  $T=0$ . A systematic study of the variation of a response function with the noise is not available here due to the inherent complexities of the dynamical system [2]; e.g., tunneling is enhanced for high and low frequency limits but it may altogether be destroyed (coherent destruction of tunneling) in the intermediate range. At very low temperatures, where quantum fluctuations still dominate, investigation on quantum stochastic resonance (QSR) is effectively reduced to the study of the dissipative dynamics of a periodically driven spin-boson system [1,4]. The noise here is characterized by the temperature of the thermal bath and by the coupling of the bistable system to the environment.

Recent results in some classical systems have shown that the positive role of noise to enable response to a signal can by no means be restricted to temporal fields only and is more universal in nature [5,6]. Even when the field is spatially periodic, it can induce spatial modulations in a bistable system with the help of noise.

We report here that one can achieve QSR at  $T=0$  in a different quantum system, in which the response can be studied as a function of the quantum noise. Motivated by the occurrence of spatial SR in classical systems, we take the external field to be periodic in space in the quantum case.

Our system is a periodically driven Ising model in a transverse field at zero temperature. The strength of the transverse field is a measure of the quantum fluctuations and can be tuned.

Although it occurs in both cases, the mechanisms of SR in fields periodic in time and spatially modulated fields are intrinsically different. In stochastic resonance in presence of time dependent fields, there are resonant transitions between the two potential wells of the bistable system. It is achieved when the time period of the temporal field is comparable to twice the system's own time scale of transitions between the neighboring potential wells, the rate of which is given by the Kramer's rate. In presence of a spatially periodic field, on the other hand, resonance implies transitions to a state with spatial correlations commensurate to those of the external field. The symmetric double well potential gets distorted differently in the two cases, although fluctuation induced transitions are responsible for resonance in both. In the spatially periodic field, the system reaches an equilibrium and studying the static properties are sufficient. Instead of time scales as in time dependent fields, one has to compare the length scales in the spatially modulated field. The field tends to form domains of size of the order of half of its wavelength. At resonance, the correlation length of the equilibrium state has to be comparable to half the spatial periodicity or wavelength of the field.

The lattice periodicity plays an important role in a spatially periodic field. The wavelength of the field may be either commensurate or incommensurate to the lattice periodicity. In the present study, we consider the commensurate case only. The case of incommensurate field has not been studied for the classical case even.

The transverse Ising system in a spatially periodic field can be described by the Hamiltonian

$$H = -J \sum_{i=1}^N S_i^z S_{i+1}^z - \sum_{i=1}^N h_i S_i^z - \Gamma \sum_i S_i^x. \quad (1)$$

Here  $h_i$  is the spatially modulated field and  $\Gamma$  the transverse field. The form of  $h_i$  is like this:  $h_i = h_0$  at  $i = (n\lambda + 1)$  to  $(2n + 1)\lambda/2$  ( $n = 0, 1, \dots, N/\lambda - 1$ ) and  $-h_0$  elsewhere. The wavelength of the field is denoted by  $\lambda$ . In principle the modulated field can be chosen to be of any form, for simplicity we choose a square wave form. The Hamiltonian (1) is comparable to that of the transverse Ising model in a field periodic in time [7,8] which is studied in the context of quan-

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tum dynamical phenomena. We consider a spatial equivalent and will be commenting on the comparable features later in this paper.

In the absence of the longitudinal field, the system has a quantum critical point at  $\Gamma/J=1$  [9]. With the field, which is competing in nature, the phase transitions in the  $\Gamma-h_0$  plane have been recently studied [10]. The system shows a continuous order-disorder phase transition from  $\Gamma/J=1$  at  $h_0=0$  to  $\Gamma/J=0$  at  $h_0=4J/\lambda$ . Beyond this value of  $h_0$ , the system is field dominated and no phase transition can occur here. Thus, our study will be limited to the region  $h_0 < 4\lambda/J$ , beyond which the system responds to the field spontaneously.  $\Gamma=0, h_0=4J/\lambda$  is a multiphase point. The model has the additional feature that the competing field and the tunneling field scale similarly close to criticality.

We obtain the average correlations between the local fields and spin components in the longitudinal ( $z$ ) direction:

$$g(h_0, \Gamma) = \frac{1}{N} \left[ \frac{1}{h_0} \sum_i \langle S_i^z h_i \rangle \right], \quad (2)$$

where  $\langle \dots \rangle$  is the expectation value and consider it as the response function [5].  $g(h_0, \Gamma) \leq 1$  by definition. Resonance will imply a maximum value of  $g(h_0, \Gamma)$ .

Stochastic resonance is a process induced by fluctuation. In a system which undergoes a continuous phase transition, the fluctuations are maximum at the critical point. However, SR is expected away from criticality where fluctuation is lesser. This is because at the critical point, the correlation length diverges and all other length scales are irrelevant. But for spatial SR to occur, the spin correlation length should equal the imposed spatial modulation. The latter being finite, SR will occur in a regime away from the critical point. This is confirmed in our results.

We conduct a numerical study by diagonalizing the Hamiltonian matrix for finite chains using Lanczos method. The system sizes are restricted to multiples of the wavelength  $\lambda$  of the field. We obtain results for system sizes  $N \leq 18$  with periodic boundary conditions. In Fig. 1, we show how the spins orient spatially (the  $z$  components of the spins are shown, to which the external periodic field is coupled) as the noise is increased for a constant value of  $h_0$ . It is interesting to note that even when the fluctuations are small so that a ferromagnetic order exists, the local magnetization shows a modulation with wavelength  $\lambda$  about a nonzero value. Beyond the ferromagnetic phase, it oscillates about zero. The modulations will increase with the noise, as expected and then decrease beyond a certain point.

In Fig. 2, we show how the response function  $g(h_0, \Gamma)$  behaves with the increasing quantum fluctuation ( $\Gamma/J$ ): it shows a maximum at a certain value of  $\Gamma = \Gamma_R$ . The position of the maximum and the value of  $g$  depend on the wavelength as well as on the field strength.

The behavior of  $\Gamma_R/J$  with the field  $h_0/J$  is shown in Fig. 3 for different values of the wavelength.  $\Gamma_R$  is not defined for  $h_0=0$  and  $\Gamma_R=0$  for  $h_0/J \geq 4/\lambda$  where the system naturally orients along the field. It may be noted that as the wavelength is increased, the resonance occurs at lower values of  $\Gamma$ . This is quite easy to understand: for increasing wave-

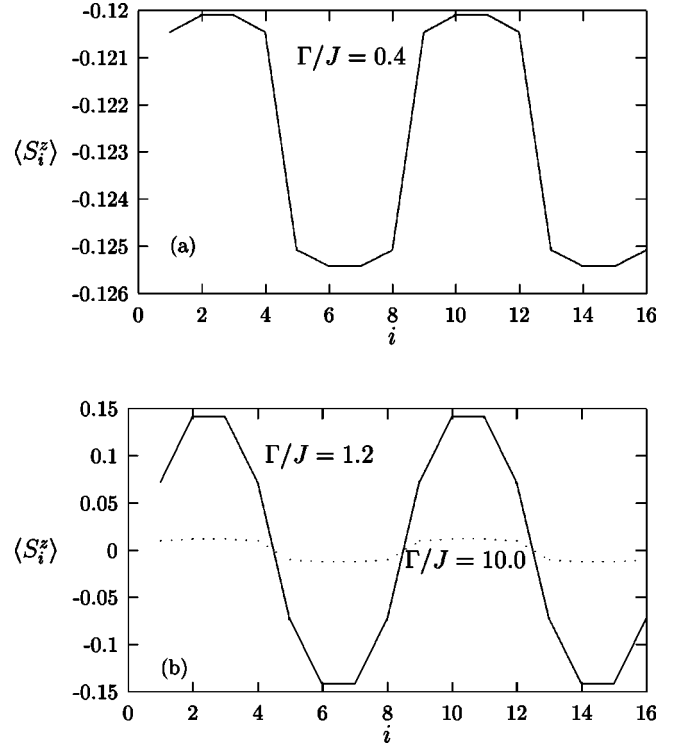


FIG. 1. Local magnetization shows an oscillatory behavior about (a) a nonzero value for small fluctuations, and (b) zero for large fluctuations. The amplitude of the oscillations decreases after reaching a maximum. The data are shown for a  $N=16$  system with  $\lambda=8$  and  $h_0/J=0.1$ .

length  $\lambda$ , one needs to create smaller number of domain walls and hence lesser amount of fluctuation is needed. The phase boundaries  $\Gamma_c(h_0)$  show opposite behavior as with increasing  $\lambda$ , one needs larger amount of fluctuations to destroy the order [10]. For a comparison of  $\Gamma_c$  and  $\Gamma_R$ , we have also shown the phase boundary line for  $\lambda=2$  in Fig. 3.

Another quantity which we study is the value of the parameter  $g(h_0, \Gamma)$  at  $\Gamma_R$ . This shows a very interesting behavior. It varies linearly with  $h_0$  up to a limiting value and then shows a nonlinear behavior, increasing faster than a

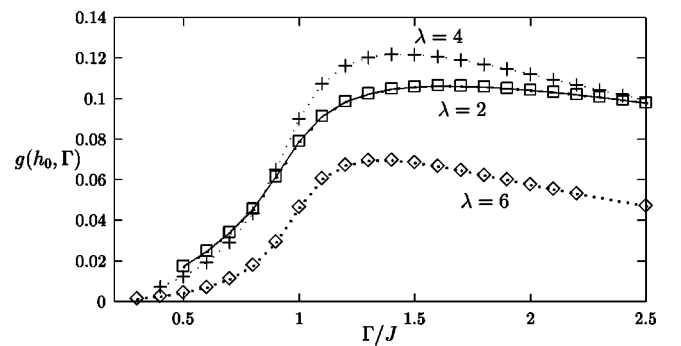


FIG. 2. Typical behavior of the response to a spatially periodic external field as a function of the quantum fluctuations for different wavelengths are shown. The values of  $h_0/J$  are small ( $\sim 10^{-1}$ ) and different for each of the curves and no systematics is expected. The maximum value of  $g$  indicates the resonance.

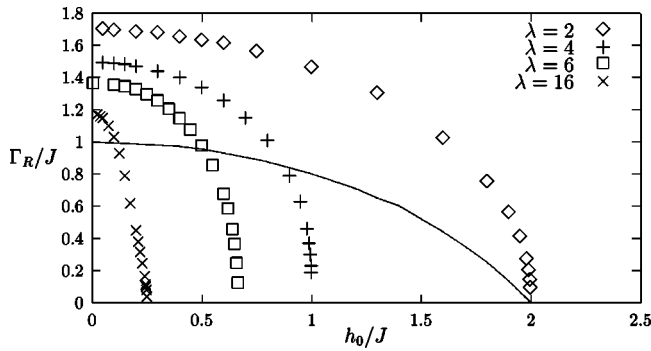


FIG. 3.  $\Gamma_R/J$  lines for different wavelengths. The values of  $h_0/J$  where  $\Gamma_R/J \rightarrow 0$  correspond to  $h_0/J = 4\lambda$ . The solid line denotes the phase boundary  $\Gamma_c$  for  $\lambda = 2$ .

power law (Fig. 4). This can be interpreted as the existence of a limiting value of  $h_0$  beyond which the field can no longer be assumed to be weak. This limit also decreases with  $\lambda$ . Beyond this limit, the unperturbed energy landscape picture seems to get drastically modified. It is also interesting to note that even for strong fields, while the maximum response is less than 0.6 for  $\lambda = 2$  and 4, it rapidly approaches unity for higher values of  $\lambda$ .

In order to study how the value of  $\Gamma_R/J$  varies with the wavelength, we plot  $\Gamma_R/J$  against  $\lambda$  for the same value of  $h_0\lambda/J$ . The reason for keeping  $h_0\lambda/J$  constant is that  $h_0\lambda$  determines the limiting value of  $h_0$  and could be used as a standard to compare results corresponding to different values of  $\lambda$ . The results are shown in Fig. 5. For values of  $h_0\lambda/J$  much lesser than 4,  $\Gamma_R/J$  shows a monotonic behavior. For larger values close to the multiphase point, there is an anomaly and the monotonic behavior is lost in the sense  $\Gamma_R/J$  first increases with  $\lambda$  and regains its decreasing nature beyond  $\lambda = 4$ . Our explanation for the above is that for very low value of  $\lambda$ , the field amplitude is very strong at high values of  $h_0\lambda$  and thus one needs a smaller amount of fluctuation to achieve resonance. This is again a feature of the strong field regime. Perhaps  $h_0\lambda/J$  is not the proper parameter to keep constant and compare results here. Apparently,  $\Gamma_R/J$  decays exponentially with  $\lambda$  for large  $\lambda$  values in the weak field regime.

At the multiphase point, the domain sizes are multiples of  $\lambda/2$ . Close to the multiphase point, one could expect corre-

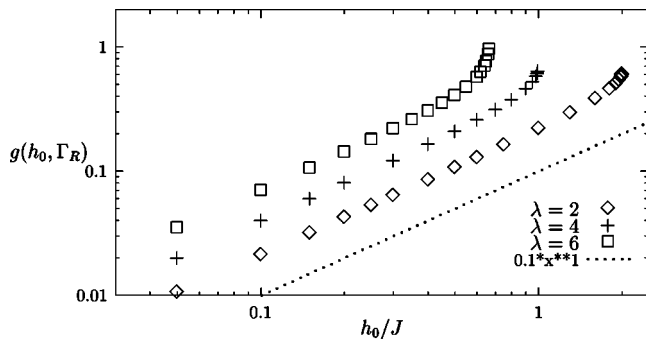


FIG. 4. Maximum value of the response is shown against the amplitude of the field. It shows a linear behavior up to a limiting value of the field.

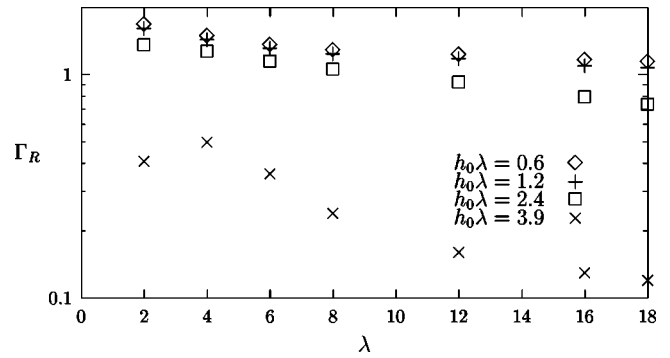


FIG. 5. Values of  $\Gamma_R/J$  are shown against  $\lambda$  keeping  $h_0\lambda/J$  constant in order to compare the results. The behavior is nonmonotonic for large  $h_0\lambda/J$ .

lations other than that of the field to manifest. We investigate this for high values of  $h_0$  close to  $4J/\lambda$  but fail to detect any other modulations barring that of  $\lambda$ . Thus, any nontrivial behavior of the magnetization (e.g., steplike structures, etc.), is definitely a feature unique to time dependent fields [7,11], as was emphasized in Ref. [7] and absent for spatially periodic fields.

The results shown in Figs. 1–5 are for  $N=12$  for  $\lambda = 12$ ,  $N=16$  for  $\lambda$  values 2, 4, 8, and 16 and  $N=18$  for  $\lambda = 6, 18$ . The finite size effects are negligible and not shown.

The general features of the spatial stochastic resonance occurring in the quantum Ising model at zero temperature are qualitatively similar to those of the classical Ising model [5]. The comparison of  $\Gamma_R$  and  $\Gamma_c$  is a unique feature of the quantum system, as no finite temperature phase transition exists in the classical one-dimensional model. However, stochastic resonance does not require that the system should undergo a phase transition. Also, it is not imperative that SR in one-dimensional quantum system and two-dimensional classical system be identical as SR is not a critical phenomena.

In summary, we have studied the phenomenon of spatial stochastic resonance in a quantum model at  $T=0$  and found that SR can be realized in this model where the fluctuations are entirely quantum in nature. The response of the system depends on the field amplitude and the wavelength. Resonance is achieved at values of the transverse field higher than its critical value where the order-disorder transition occurs. The maximum response behaves differently in the so-called weak field and strong field regimes. We could not detect any oscillations of the local magnetization with periodicity different from that of the external field even close to the highly degenerate multiphase point.

The reason we could observe quantum stochastic resonance at zero temperature are twofold: first, the quantum model we consider may be special. Second, the spatial and temporal fields signify entirely different processes of stochastic resonance. Whether it is possible to obtain QSR in a field periodic in time for the transverse Ising model will be an interesting study and confirm whether the first conjecture is correct. The ground states of the transverse Ising model and the quantum double well system are equivalent and the counterintuitive feature like coherent destruction of tunnel-

ing is present in both [2,7] when a field periodic in time is present. Also, a simple picture such as Fig. 2 is unlikely to exist in the presence of a time dependent periodic field, as there are indications of additional nontrivial oscillations of the magnetization in quantum spin models [7,11]. Hence, it seems that choosing the field periodic in space, rather than considering a specific quantum system, is responsible for observing quantum stochastic resonance analogous to the classical case.

Although we are interested in the static results only for

SR, there will be an inherent quantum dynamics in the model. The dynamics of domain growth could be an interesting study which in this case may be called a local nucleation phenomena; the domains will be finite in size due to the presence of the spatially periodic external field.

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